MATH 6337 HOMEWORK #2 Due date: September 11, 2007

Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

1. Problem 1.3 #6. Complete the proof of Theorem 1.9.

2. Let Z be any subset of $\mathbb{R}$ with Lebesgue measure zero. Show that the set $\{x^2 : x \in Z\}$ also has Lebesgue measure zero.

3. Let E_1 be a Lebesgue measurable subset of $\mathbb{R}^m$ and let E_2 be a Lebesgue measurable subset of $\mathbb{R}^n$. Prove that $E_1 \times E_2 = \{(x, y) : x \in E_1, y \in E_2\}$ is a Lebesgue measurable subset of $\mathbb{R}^{m+n}$, and that $|E_1 \times E_2| = |E_1|\,|E_2|$.

Hint: First do the following special cases: a. both E_1, E_2 are open, b. both E_1, E_2 are G_δ-sets, c. either E_1 or E_2 has measure zero.

4. Show that there exist disjoint $E_1, E_2, \ldots \subseteq \mathbb{R}$ such that

$$\left|\bigcup_{k=1}^{\infty} E_k\right|_e < \sum_{k=1}^{\infty} |E_k|_e,$$

with strict inequality.

Hint: Let E be a nonmeasurable subset of $[0, 1]$ whose rational translates are disjoint. Consider the translates of E by all rational $r \in (0, 1)$, and use the fact that exterior Lebesgue measure is translation-invariant.