You can use your book and notes. No laptop or wireless devices allowed. Write clearly and try to make your arguments as linear and simple as possible. The complete solution of one exercise will be considered more that two half solutions. In your solution you can use only statements that were proven in class.

Name: ______________________________

Question:	1	2	Total
Points:	30	30	60
Score:			

1. Consider the equation for $x \in \mathbb{R}^n$

$$\dot{x} = -x + g(x) \tag{1}$$

where

$$\|g(x)\| \leq C\|x\|^2.$$

for some $C > 0$.

Let $x(t)$ the soltion of eq.(1) such that $x(0) = x_0$.

(a) (10 points) Show that if $\|x_0\| \leq 1/C$ than $\|x(t)\| \leq \|x_0\|$ for every t.(**Hint:** write a differential equation for $\|x(t)\|$.)

(b) (10 points) Use eq.(A4-4) page 253 from the book to show that, for $\epsilon \leq 1$, if $\|x_0\| \leq \epsilon/C$ than

$$\|x(t)\| \leq e^{-t}\|x_0\| + \epsilon \int_0^t e^{-(t-s)}\|x(s)\|ds.$$

(c) (10 points) Conclude that forevery ϵ there exists δ such that

$$\|x(t)\| \leq e^{-(1-\epsilon)t}\|x_0\|$$

it $\|x(0)\| \leq \delta$. (**Hint:** Call $y(t) = e^t\|x(t)\|$. Use Gronwall Lemma and point 2) to show that $y(t) \leq y(0)e^{\epsilon t}$.)

2. Consider the equation in $\mathbb{R}^2$

$$\dot{x} = E - \frac{(E \cdot x)}{(x \cdot x)} x \tag{2}$$

where $(x \cdot y) = x_1 y_1 + x_2 y_2$ and $E \in \mathbb{R}^2$.

(a) (10 points) Show that $H(x) = (x \cdot x)$ is an first integral for eq.(2).

(b) (10 points) Find all fixed point of eq.(2) and discuss their stability.

(c) (10 points) Are there periodic orbit?