Fall 05 Name: ______________________________

Math 4581 Test 2 retake Bonetto

1) The equation governing the temperature $u(x,t)$ inside a rod is:

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = \dfrac{\partial^2 u(x,t)}{\partial x^2} \qquad 0 \leq x \leq 1 \\ \dfrac{\partial u(0,t)}{\partial x} = 0 \\ \dfrac{\partial u(1,t)}{\partial x} = r\,(T - u(1,t)) \\ u(x,0) = Tx \end{cases}$$

a) write and solve the equation for the steady state $v(x)$.

b) write the equation for the difference $w(x,t) = u(x,t) - v(x)$.

c) use separation of variable to reduce the problem to a Sturm-Luiville problem. Find the eigenvalues and eigenfunctions. Explain why you can expand in eigenfunctions. Write the general solution for $w(x,t)$ and an expression for the coefficient in term of $w(x,0)$.

d) write the solution of the problem. Remember that

$$\int x\cos(\lambda x)dx = \frac{\cos(\lambda x)}{\lambda^2} + \frac{x\sin(\lambda x)}{\lambda}$$

$$\int \cos^2(\lambda x)dx = \frac{x}{2} - \frac{\sin(2\lambda x)}{4\lambda}$$

e) Consider the equation:

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = \dfrac{\partial^2 u(x,t)}{\partial x^2} + k(T - u(x,t)) \qquad 0 \le x \le 1 \\ \dfrac{\partial u(0,t)}{\partial x} = 0 \\ \dfrac{\partial u(1,t)}{\partial x} = r\,(T - u(1,t)) \\ u(x,0) = Tx \end{cases}$$

Find the solution of this equation. Observe that after comnputing the steady state you can just use the results in points a), b) and c). What is the only difference with the previous case?