No books or notes allowed. No laptop or wireless devices allowed. Write clearly.

Name: ______________________________

Question:	1	2	3	Total
Points:	30	40	40	110
Score:				

Question 1 . . . . *30 point*

Let T_1 and T_2 be two independent exponential r.v. with parameter 1. This means that the p.d.f. $f(t_i)$ of T_i is

$$f(t) = \begin{cases} e^{-t} & \text{if} \quad x_i \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, 2$. Moreover let N be a Poisson r.v. with parameter t. This means that the p.m.f. $p(n)$ of N is

$$p(n) = \frac{t^n}{n!} e^{-t}$$

(a) (10 points) Compute the probability that $T_1 + T_2 < 1$, *i.e.* $P(T_1 + T_2 < 1)$.

(b) (10 points) Show that $P(T_1 < t) = P(N \geq 1)$.

(c) (10 points) Show that $P(T_1 + T_2 < t) = P(N \geq 2)$.

Question 2 .... *40 point*

You decide to go to Las Vegas to play roulette. You select at random one number between 1 and 36 and bet \$1 on that number. The outcome of the roulette is a number between 0 and 36, *i.e.* there are 37 possible outcomes. They all have the same probability. If the outcome is equal to the number you selected, you get back \$36, *i.e.* you win \$35. If not you lose your dollar. Let X be the r.v. that describe your win (or loose).

(a) (10 points) Compute the p.m.f. of X, its expected value $E(X)$ and its variance $V(X)$.

(b) (10 points) Suppose that in that evening you play 300 times in the same way, every time selecting the same number. Let Y be the total amount of money you win (or loose) during the evening. Give the (approximate) p.d.f. of Y. Use the C.L.T.

(c) (10 points) If at the beginning of the evening you had \$100, what is the probability that, at the end of the evening you have more than \$100? and the probability that you have some money left?

(d) (10 points) Do the above answers change if, at every game, you select randomly the number to bet on? Why?

Question 3 .... *40 point*

Let X be a normal r.v. with $\mu_X = 3$ and $V(X) = 12$ and Y be a normal r.v. with $\mu_Y = 2$ and $V(Y) = 3$. Assume that X and Y are independent. Let $Z = X - 2Y + 1$

(a) (10 points) Compute $P(Z > 5)$.

(b) (10 points) Find the value of c such that $P(-c < Z < c) = 0.7$.

(c) (10 points) Assume now that $\mathrm{Cov}(X, Y) = 4$ and that Z is still a normal r.v. Compute $P(Z > 5)$.

(d) (10 points) Assume now that $\mathrm{Cov}(X, Y) = 6$ and that Z is still a normal r.v. Compute $P(-1 < Z < 1)$. Explain your result.