n 27

a) $c = 2.14$

b) $\phi(c) - \phi(0) = 0.291$

$\phi(c) = 0.791$

$c = 0.81$

c) $1 - \phi(c) = 0.121$

$\phi(c) = 0.879$

$c = 1.17$

d) $2\phi(c) - 1 = 0.668$

$\phi(c) = 0.834$

$c = 0.97$

e) $2\phi(-c) = .016$

$\phi(-c) = 0.008$

$c = 2.41$

n 30

a) $\Phi\left(\frac{100-80}{10}\right) = \phi(2) = 0.9772$

b) $\phi(0) = 0.5$

c) $\cancel{\phi\left(\frac{65-80}{}\right)}$ $\phi(2) - \phi\left(\frac{65-80}{10}\right) =$

$= \phi(2) - \phi(-1.5) = 0.9772 - 0.0668$

d) $= 1 - \phi\left(\frac{70-80}{10}\right) = 1 - \phi(-1) = 1 - 0.1587$

33

a) $1-\phi\left(\frac{10-8.8}{2.8}\right)=1-\phi\left(\frac{1.2}{2.8}\right)=0.3336$

b) $\phi\left(\frac{20-8.8}{2.8}\right)=\phi\left(\frac{11.2}{2.8}\right)\simeq 0$

c) $\phi\left(\frac{1.2}{2.8}\right)-\phi\left(\frac{3.8}{2.8}\right)=0.5795$

d) $P(8.8-c \leq X \leq 8.8+c)=$

$\phi\left(\frac{+c}{2.8}\right)-\phi\left(\frac{-c}{2.8}\right)=2\phi\left(\frac{c}{2.8}\right)-1=0.98$

$\phi\left(\frac{c}{2.8}\right)=0.99$ $\qquad \frac{c}{2.8}=2.33$

$c=6.524$

e) $1-P(\text{none exceeds } 10\text{in})=$

$1-(0.336)^4$

n 37

$$P(X \leq 100m) = \Phi\left(\frac{100-200}{30}\right) =$$

$$= \Phi(-0.33) = 0.3707$$

P(AT least one demage) =

$$1 - P(\text{no demage}) = 1 - (1-0.3707)^5$$

n 38

$$P(X > 10.256) = 0.1 \quad = 1 - \phi\left(\frac{-\mu + 10.256}{\sigma}\right) = 0.1$$

$$P(X < 9.671) = 0.05 \quad = \phi\left(\frac{-\mu + 9.671}{\sigma}\right) = 0.05$$

$$\Phi\left(\frac{-\mu + 10.256}{\sigma}\right) = 0.9 \quad \cancel{\Rightarrow \mu = 10.256 - 0.128}$$

$$\phi\left(\frac{-\mu + 9.671}{\sigma}\right) = 0.05 \quad \cancel{\mu = 9.671}$$

$$-\mu + 10.256 = 1.28\,\sigma$$

$$-\mu + 9.671 = -1.645\,\sigma$$

$$\Downarrow$$

$$\sigma = \frac{0.588}{2.925} = 0.2 \qquad \mu = 10$$

n 42

a) $\Phi\left(\frac{75-70}{3}\right) - \phi\left(\frac{67-70}{3}\right) =$

$\phi(1.66) - \phi(-1) = 0.9515 - 0.1587 = 0.7928$

b) $2\phi\left(\frac{c}{3}\right) - 1 = 0.95$

$\phi\left(\frac{c}{3}\right) = 0.975$

$\frac{c}{3} = 1.96$

$c = 5.88$

c) Expected # $= n \cdot p = 10 \cdot 0.7928 = 7.928$

d) $p = \phi\left(\frac{73.84-70}{3}\right) = \phi(1.28) = 0.8997$

Prob $= 1 - (1-0.8997)^{10} - 10 \cdot 0.8997 \cdot (1-0.8997)^{9}$

51

X = # driver That wear seal belt out of 500

X = Normal $200, 500 \cdot \sqrt{0.24} = \sigma$

$$\text{Prob} = \phi\left(\frac{230-200}{\sigma}\right) - \phi\left(\frac{180-200}{\sigma}\right)$$

n 59

a) $E(X) = \frac{1}{\lambda} = 1$

b) $V(X) = \frac{1}{\lambda^2} = 1$

c) $P(X \leq 4) = F(4) = 1 - e^{-4}$

d) $P(2 \leq X \leq 5) = F(5) - F(2) = 1 - e^{-5} - 1 + e^{-2} =$
$= e^{-2} - e^{-5}$

n 61

a) I the mean is 25.000 then $\lambda = \frac{1}{25.000}$

$P(X \geq 20.000) = e^{-\frac{20.000}{25.000}} = e^{-0.8}$

$P(X \leq 30.000) = 1 - e^{-\frac{30.000}{25.000}} = 1 - e^{-1.2}$

$P(20.000 \leq X \leq 3000) = 1 - e^{-1.2} - 1 + e^{-0.8} =$
$e^{-0.8} - e^{-1.2}$

b)

n 63

a) $\{X > t\} = A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5$

b) $P(X > t) = P(A_1)P(A_2)P(A_3)P(A_4)P(A_5) =$

$e^{-5\lambda t}$

$F(t) = 1 - e^{-5\lambda t}$

$f(t) = 5\lambda e^{-5\lambda t}$ exp. par $5\lambda = 0.05$

c) Same reasoning

$f(t) = n\lambda\, e^{-n\lambda t}$ exp par $n\lambda = n \cdot 0.01$

n 1

a) 0.20

b) $0.1 + 0.04 + 0.08 + 0.2 = 0.42$

c) There is at least 1 hose in use at each island

$0.2 + 0.06 + 0.14 + 0.30 = 0.7$

d) $p_X(0) = 0.16$ $\quad p_X(1) = 0.34$ $\quad p_X(2) = 0.5$

$p_Y(0) = 0.24$ $\quad p_Y(1) = 0.38$ $\quad p_Y(2) = 0.38$

$P(X \leq 1) = 0.16 + 0.34 = 0.5$

e) $P(X=0)P(Y=0) = 0.16 \cdot 0.24 \neq P(X=0, Y=0) = 0.1$

Not independent

n 3

a) 0.15

b) 0.08 + 0.15 + 0.10 + 0.07 = 0.4

c) $P(|X_1 - X_2| \geq 2)$

0.05 + 0.03 + 0.01 + 0.04 + 0.05 + 0.04 = 0.22

d) Exactly 4

0.03 + 0.1 + 0.04 = 0.17

Exactly 5

0.01 + 0.04 + 0.06 = 0.11

Exactly 6

0.05 + 0.07 = 0.12

Exactly 7 0.06

At least 4

0.17 + 0.11 + 0.12 + 0.06 = 0.46

n. 8

a) $$\frac{\binom{8}{3}\binom{10}{2}\binom{12}{1}}{\binom{30}{6}} =$$

b) $$p(x,y) = \begin{cases} \dfrac{\binom{8}{x}\binom{10}{y}\binom{12}{6-x-y}}{\binom{30}{6}} & x+y < 6 \\ 0 & \text{otherwise} \end{cases}$$

n 12

a) $$P(X > 3) = \cancel{\int_{-\infty}^{\infty}\int_{3}^{3} x e^{-x(1+y)}\, dx}$$

$$= \int_3^\infty \int_0^\infty x e^{-x(1+y)}\, dy\, dx =$$

$$\int_3^\infty x e^{-x} \int_0^\infty e^{-xy}\, dy\, dx =$$

$$\int_3^\infty e^{-x}\, dx = e^{-3}$$

b) $f_X(x) = \int_0^\infty x e^{-x(1+y)} \, dy = e^{-x}$

$f_Y(y) = \int_0^\infty x e^{-x(1+y)} \, dx = \frac{1}{(1+y)^2}$

c) $1 - \int_0^3 \int_0^3 x e^{-x(1+y)} \, dx \, dy$

n 15

a) $$P(Y \geqslant y) = P(X_1 \geqslant y) \cdot \left(P(X_2 \geqslant y \text{ or } X_3 \geqslant y) \right) =$$

$$P(X_1 \geqslant y) \cdot \left(1 - P(X_2 < y) \cdot P(X_3 < y) \right) =$$

$$= e^{-\lambda y} \left(1 - (1 - e^{-\lambda y})(1 - e^{-\lambda y}) \right) =$$

$$e^{-\lambda y} \left(2e^{-\lambda y} - e^{-2\lambda y} \right)$$

$$F(y) = 1 - P(Y \geqslant y) = 1 - 2e^{-2\lambda y} + e^{-3\lambda y}$$

b) $$f(y) = 4\lambda e^{-2\lambda y} - 3\lambda e^{-3\lambda y} \qquad y > 0$$

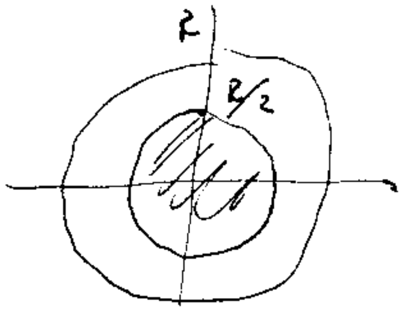
$$\int_0^\infty y f(y)\, dy = 2 \int_0^\infty 2\lambda y e^{-2\lambda y}\, dy -$$

$$\int_0^\infty 3\lambda y e^{-3\lambda y}\, dy =$$

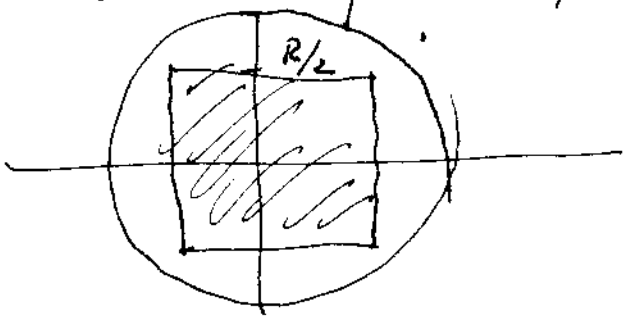
$$= \frac{2}{2\lambda} - \frac{1}{3\lambda} = \frac{2}{3} \frac{1}{\lambda}$$

n 17

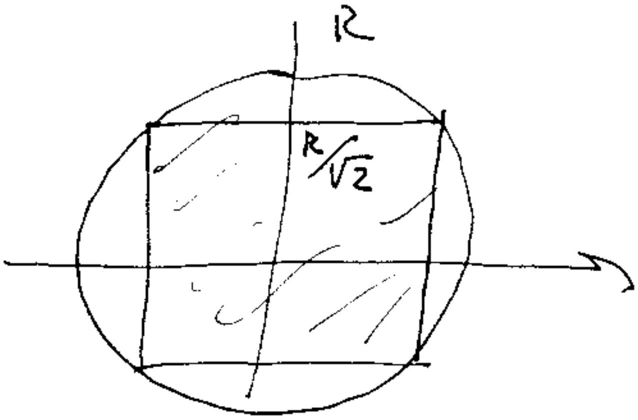
a) $$\int_{x^2+y^2\le \frac{R^2}{4}} \frac{1}{\pi R^2}\, dx\, dy = \frac{1}{\pi R^2}\, \pi \frac{R^2}{4} = \frac{1}{4}$$



b) R^2 = area of the square of side R

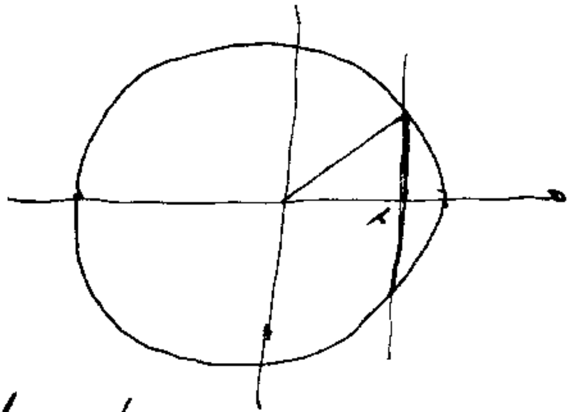


c) $2R^2$



d $$f_X(x) = \frac{2\sqrt{R^2 - x^2}}{\pi^2 R^2}$$

$$f_Y(y) = \frac{2\sqrt{R^2 - y^2}}{\pi^2 R^2}$$



Clearly not independend

$$f_Y(x)\, f_Y(y) \neq \frac{1}{\pi^2 R^2}$$

n 20

a) $$f_{X_3 | X_1 X_2}(x_3 | x_1, x_2) = \frac{f(x_1, x_2, x_3)}{f_{X_1 X_2}(x_1, x_2)}$$

where

$$f_{X_1 X_2}(x_1, x_2) = \int_{-\infty}^{\infty} f(x_1, x_2, x_3)\, dx_3$$

b) $$f_{X_2 X_3 | X_1}(x_2, x_3 | x_1) = \frac{f(x_1, x_2, x_3)}{f_{X_1}(x_1)}$$

where

$$f_{X_1}(x_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{-\infty} f(x_1, x_2, x_3)\, dx_2\, dx_3$$

n 22

a) $0 \cdot 0.2 + 5 \cdot (0.6 + 0.4) + 10(0.01 + 0.15 + 0.02) +$
$15 \cdot (0.15 + 0.20 + 0.10) + 20(0.14 + 0.10) +$
$+ 25 \cdot 0.01$

b) $0 \cdot 0.02 + 5(0.06 + 0.04 + 0.15) +$
$+ 10 \cdot (0.02 + 0.20 + 0.14 + 0.15 + 0.01) +$
$+ 15 \cdot (0.10 + 0.10 + 0.01)$

n 25 Area of rectangle $= xy$

$E(xy) = E(x)E(y) = L^2$

n 27

$$\int_0^1 \int_0^1 3x^2\, 2y\, |x-y|\, dx\, dy =$$

$$\int_0^1 \int_x^1 3x^2\, 2y\, (y-x)\, dy\, dx \;+$$

$$\int_0^1 \int_y^1 3x^2\, 2y\, (\cancel{x}-y)\, dy\, dx = \frac{1}{3}$$

~~n 30~~ n 30

Compute

$$E(XY) = \sum_x \sum_y xy\, p(x,y)$$

$$E(X) = \sum_x \sum_y x\, p(x,y)$$

$$E(Y) = \sum_x \sum_y y\, p(x,y)$$

$$E(X^2) = \sum_x \sum_y x^2 p(x,y) \qquad E(Y^2) = \sum_x \sum_y y^2 p(x,y)$$

Then

$$\sigma_x^2 = V(X) = E(X^2) - E(X)^2 \qquad \sigma_y = V(Y) = E(Y^2) - E(Y^2)$$

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

b) $\rho = Cov(X,Y) / \sigma_x \sigma_y$

n 36

$$E(aX+b) = aE(X)+b$$

$$\sigma^2_{aX+b} = V(aX+b) = a^2 V(X)$$

$$E((aX+b)X) = aE(X^2) + bE(X)$$

$$Cov(aX+b, X) = E((aX+b)X) - E(aX+b)E(X) =$$

$$aE(X^2) \cancel{+bE(X)} - aE(X)^2 \cancel{-bE(X)} =$$

$$aV(X)$$

$$\rho = \frac{Cov(aX+b, X)}{\sigma_X \, \sigma_{aX+b}} = \frac{a\,V(X)}{\sqrt{V(X)}\sqrt{a^2 V(X)}} = \frac{a}{|a|} =$$

sign a

$\rho = 1$ if $a > 0$.