Fall 04 Name: ______________________________

Math 3770 Test 2 Bonetto

1) In a bowl there are 3 balls numbered form 1 to 3. You extract two of them without reinsertion. Let $X_1$ the result of the first extraction and $X_2$ the result of the second one. Compute:

   a) the joint probability mass function $p(x_1, x_2)$ of $X_1$ and $X_2$.

   b) the probability mass function of $Y = X_1 + X_2$ and the expected value $E(X_1 + X_2)$.

   c) the marginal $p_{X_1}(x_1)$ with respect to $X_1$ of $p(x_1, x_2)$ and the conditional probability mass function $p_{X_1|X_2}(x_1|x_2)$.

   d) $\text{cov}(X, Y)$. Are $X_1$ and $X_2$ independent? Why?

2) Let $X$ be an exponential rv with parameter $\lambda = 2$ and $Y$ be a uniform rv between 0 and 1. Assume that $X$ and $Y$ are independent. Compute:

   a) the joint probability distribution function $f(x, y)$ of $X$ and $Y$.
   b) the probability that $X > 3$ and $Y > 0.5$
   c) the probability that $X$ is larger than $Y$, that is $P(X > Y)$.

   **(Bonus)** Assume now that $X$ and $Y$ are not independent but $X$ is still exponential with parameter $\lambda = 2$ while the conditional probability of $Y$ given $X$ is $f_{Y|X}(y|x) = 1/x$ for $0 < y < x$ and $f_{Y|X}(y|x) = 0$ otherwise. Compute

   d) the joint probability distribution function $f(x, y)$ of $X$ and $Y$.
   e) the covariance of $X$ and $Y$, $\text{cov}(X, Y)$.
   (**Hint**: remember that:

$$\int_0^\infty \lambda x e^{-\lambda x} = \frac{1}{\lambda} \qquad \int_0^\infty \lambda x^2 e^{-\lambda x} = \frac{2}{\lambda^2}$$

   ).

3) You reach campus driving while a friend of yours use public transportation. The time you need to reach campus is described by a rv $X$ with $E(X) = 1h$ and standard deviation $\sigma_X = 0.3h$ while the time needed by your friend is a rv $Y$ with $E(Y) = 1.2h$ and standard deviation $\sigma_Y = 0.5h$. In a semester you both go to campus 100 times. Moreover the times needed to reach campus on different days are independent. Let $T_x$ be the total time you spend driving during a semester.

   a) What is the (approximate) distribution of $T_X$? Compute $P(T_X > 110)$.

   Call $\bar{X}$ the average time (over the semester) you spend driving to campus and $\bar{Y}$ the average time your friend spend in the public transportation system.

   b) compute $P(\bar{X} > 1.05)$ and $P(1.1 < \bar{Y} < 1.3)$.
   c) compute $P(\bar{X} > \bar{Y})$.