No books or notes allowed. No laptop or wireless devices allowed. Write clearly.

Name: ____________________

Question:	1	2	3	4	5	Total
Points:	40	40	30	30	40	180
Score:						

Question 1 .... *40 point*

Your company assembles PCs. A new assembly line is being proposed to your attention. The following data x_i are the result of a random sample of size $N = 50$ of the production time, in minutes, of one PC using the new assembly line. They are ordered in increasing order.

26.16	26.18	26.44	26.78	26.94	27.06	27.15	27.2	27.31	27.39
27.53	27.55	27.70	27.86	28.04	28.23	28.28	28.31	28.47	28.53
28.66	28.80	28.83	28.97	28.97	29.14	29.45	29.59	29.61	29.7
29.73	29.74	29.83	30.21	30.29	30.36	30.48	30.61	30.72	31.21
31.64	31.74	31.89	32.08	32.39	32.39	32.76	32.81	32.91	32.99

You know that

$$\sum_{i=1}^{50} x_i = 1467.84 \qquad \sum_{i=1}^{50} x_i^2 = 43278.96$$

(a) (10 points) Compute the sample average and standard deviation.

(b) (10 points) Compute the median and fourth spread.

(c) (10 points) After finding eventual outliers, draw a box plot for the data.

(d) (10 points) Choose a reasonable number of classes and draw an histogram for the data, using the densities. Show your computations.

Question 2 .... *40 point*

Exercise 1 continued.

(a) (10 points) Find the Upper Confidence Bound and the Confidence Interval for the true population mean μ at a confidence level of 99%. Use the table provided to find the necessary critical values.

(b) (10 points) The assembly line you have has an average production time per PC of $\mu_0 = 30$ minutes. You want to decide weather to switch to the new assembly line. Formulate the correct null hypothesis H_0 and alternate hypothesis H_a.

(c) (10 points) You decide to hold your test at a significance level of 0.05. Give the 0.05 significance level rejection region (in term of $\bar{x}$ or z, as you prefer). Will you reject H_0 at this significance level?

(d) (10 points) Give the P-value of the above test and explain its meaning.

Question 3 . . . . *30 point*

The time difference between two consecutive arrivals of a car at a service center is described by the random variable T. The p.d.f. of this variables is:

$$f(t) = \begin{cases} \lambda e^{-\lambda(t-a)} & t > a \\ \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $a = 2$.

(a) (10 points) Compute the maximum likelihood estimator for λ for a sample of size n.

(b) (10 points) The following data are 10 observation t_i of arrival times:

18.138	2.1721	5.43	19.405	3.528	2.9496	2.3909	7.6107	7.7469	19.335

where $\bar{t} = 88.706$. Use the above result to give an estimate for λ.

(c) (10 points) Suppose now that also a is not known. Find the maximum likelihood estimator for a and λ. Estimate again a and λ from the above data. (**Hint**: Observe that the likelihood function is 0 if a is greater than any of the t_i while it increases in a for a smaller then the t_i.)

Question 4 .... *30 point*

In a company, when a customer arrives, he is assigned to one of two service facilities, facility 1 and facility 2. The service time T_1 (measured in hours) for a customer at facility 1 is an exponential random variable with parameter $\lambda = 2$ while the service time T_2 (measured in hours) for a customer facility 2 is an exponential random variable with parameter $\lambda = 4$. Assume that the customers are randomly assigned to one of the two facilities with probability $p = 0.3$ of being assigned to facility 1. Let T be the random variable that describes the service time for a costumer. (**Hint**: Remember that the p.d.f. of an exponential random variable with parameter λ is $f(x) = \lambda \exp(-\lambda x)$.)

(a) (10 points) Compute the probability that the service time T for a costumer is less the t, that is the c.d.f. T. (**Hint**: Compute the probability that $T < t$ given that the costumer is assigned to facility 1 or 2 and then use the Law of Total Probability.)

(b) (10 points) Find the p.d.f of T.

(c) (10 points) Assume now that you know that the service time for a given customer was longer that 1 hour. What is the probability that he was serviced by facility 1? (**Hint**: use Bayes Theorem in the setting of point (a))

Question 5 .... *40 point*

A food distributor carries two different brands of a certain type of grain. Its supplies fluctuate randomly in time but never exceed 2 tons. Let X be the amount of brand A on hand and Y the amount of brand B on hand. Suppose that the joint p.d.f. of X and Y is:

$$f(x,y) = \begin{cases} kxy & x > 0,\ y > 0,\ x + y < 2 \\ \\ 0 & \text{otherwise} \end{cases}$$

(a) (10 points) Draw the region where $f(x,y) \neq 0$ and find the value of k that makes $f(x,y)$ a p.d.f.

(b) (10 points) Are X and Y independent? Answer by first deriving the marginal p.d.f. of each variable.

(c) (10 points) Find $E(X)$ and $E(Y)$.