MATH 3670 Take Home Midterm Due April 23 2018 before class.

Use only your texbook. No laptop or wireless devices allowed. Write clearly.

Name: ____________________

Question:	1	2	3	4	Total
Points:	40	20	20	20	100
Score:					

Question 1 . . . . *40 point*

The water from a new natural source is bottled but must pass health inspection to be approved. In particular the bottling company must check for the average content of a particular chemical A in the bottles. The following data x_i are the result of a random sample of size $N = 50$ of the content of A in mg in the new bottles. They are ordered in increasing order.

2.02	2.04	2.05	2.12	2.17	2.25	2.26	2.33	2.36	2.37
2.39	2.46	2.49	2.49	2.58	2.64	2.71	2.76	2.79	2.84
2.93	2.94	3.06	3.07	3.09	3.09	3.11	3.26	3.29	3.32
3.34	3.37	3.43	3.45	3.46	3.47	3.51	3.51	3.55	3.64
3.64	3.70	3.77	3.84	3.85	3.85	3.86	3.87	3.90	3.98

You know that

$$\sum_{i=1}^{50} x_i = 152.27 \qquad \sum_{i=1}^{50} x_i^2 = 481.45.$$

(a) (10 points) Compute the sample average and standard deviation.

Solution:

$$\bar{x} = \frac{152.27}{50} = 3.05 \qquad s^2 = \frac{1}{49}\left(481.45 - \frac{152.27^2}{50}\right) = 0.36$$

so that

$$s = 0.60$$

(b) (10 points) Compute the median and fourth spread.

Solution:

$$m = \frac{x_{25} + x_{26}}{2} = \frac{3.09 + 3.09}{2} = 3.09$$

$$lf = x_{13} = 2.49 \qquad uf = x_{38} = 3.51 \qquad fs = uf - lf = 3.51 - 2.49 = 1.02$$

(c) (10 points) Draw a box plot for the data.

Solution:

(d) (10 points) Choose a reasonable number of classes and draw an histogram for the data, using the densities. Show your computations.

Solution:
The best choice is 7 classes. The minimum of the data can be taken as 1.95 and the maximum 4.05 so that each class has size 0.3. We thus have

Class	Freq.	Rel. Freq.	Density
1.95 - 2.25	6	0.12	0.40
2.25 - 2.55	8	0.16	0.53
2.55 - 2.85	6	0.12	0.40
2.85 - 3.15	7	0.14	0.47
3.15 - 3.45	7	0.14	0.47
3.45 - 3.75	8	0.16	0.53
3.75 - 4.05	8	0.16	0.53

Question 2 . . . . *20 point*

A baker produces every morning an amount of bread equal to the orders for that day he received the evening before. Call X this amount expressed in lbs and assume that it is an exponential r.v. with $E(X) = m$. The price of the bread is \$1/lb, that is he will get \$$x$ if he sells x lbs of bread. Due to economy of scale, the cost $c(x)$ of producing x lbs of bread is $c(x) = \sqrt{x}$. Compute the expected profit of the baker. Find the value of m for which the baker break even, that is his expected profit is \$0. (**Hint**: Call $p(x)$ the profit of the baker when he receive orders for x lbs of bread. You have to compute $E(p(X))$. See last page for useful formulas.)

Solution: The income $i(x)$ is given by:

$$p(x) = x - \sqrt(x)$$

so that

$$E(p(X)) = \int_0^\infty \frac{1}{m} e^{-\frac{x}{m}} (x - \sqrt{x}) dx = m - \frac{\sqrt{\pi m}}{2}.$$

Thus the baker break even when

$$m - \frac{\sqrt{\pi m}}{2} \qquad \Longrightarrow \qquad m = \frac{\pi}{4}.$$

Question 3 .... *20 point*

You want to use a set of wires to suspend a weight for the ceiling of your workshop.

After many tests, you reach the conclusion that each wire used has a breaking point given by a random variable X with $E(X) = 10\text{kg}$ and $\sigma_X = 2\text{kg}$.

You must suspend a weight of 500kg so you decide to use a large number of wires to suspend it. Suppose that the breaking point of a set of wires used together is the sum of the breaking points of each of them and that each wire is independent from the others.

Compute the probability the the wires will not break when you use 50, 54 or 58 of them. (**Hint**: since 50 is large, you can use the CLT.)

Solution:

If you use $N > 40$ wires, the breaking point will be a Normal r.v. Y with

$$E(Y) = 10N \qquad \sigma_Y = 2\sqrt{N}$$

You want

$$P(Y > 500) = P\left(\frac{Y - 10N}{2\sqrt{N}} > \frac{500 - 10N}{2\sqrt{N}}\right) = 1 - \Phi\left(\frac{500 - 10N}{2\sqrt{N}}\right)$$

Thus we get

N	$P(Y > 500)$
50	$1 - \Phi(0) = 0.5$
54	$1 - \Phi(-2.72) = 0.997$
58	$1 - \Phi(-5.25) = 1.0$

Question 4 . . . . *20 point*

Two discrete r.v. X and Y can take the value -1, 0 and 1. The joint p.m.f of the two variables is given by

		Y		
		-1	0	1
	-1	0.1	0.15	0.1
X	0	0.15	0.0	0.15
	1	0.1	0.15	0.1

(a) (10 points) Compute the expected values $E(X)$, $E(Y)$, variances $V(X)$, $V(Y)$ and the correlation coefficient $\rho_{X,Y}$ of X and Y.

Solution: We have that

$$p_X(-1) = p_X(1) = p_Y(-1) = p_Y(1) = 0.35 \qquad p_X(0) = p_Y(0) = 0.3$$

so that

$$E(X) = 0 \qquad V(X) = 0.7$$

Clearly also

$$E(Y) = 0 \qquad V(Y) = 0.7$$

Finally we have $E(XY) = 0$ so that

$$\rho_{X,Y} = 0$$

(b) (10 points) Are X and Y independent?

Solution:
No. For example we have

$$P(X = 0 \;\&\; Y = 0) = 0.0 \neq P(X = 0)P(Y = 0) = 0.09.$$

Useful Facts

- **A useful Integral**:

$$\int_0^\infty x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$$

- **C.L.T.**: Let X_i, $i = 1, \ldots, N$, be N independent and identically distributed r.v. Then $T_0 = \sum_{i=1}^N X_i$ is a Normal r.v.