No books or notes allowed. No laptop or wireless devices allowed. Show all your work for full credit. **Write clearly and legibly**.

Name (print): ______________________________

Question:	1	2	3	Total
Points:	40	60	0	100
Score:				

Question:	1	2	3	Total
Bonus Points:	0	10	10	20
Score:				

Question 1 .... *40 point*

The following numbers x_i, $i = 1, \ldots, 19$, represent a sample of size $n = 19$ from a given population.

-0.628	-0.993	0.970	-0.325	0.779
0.510	-0.816	-0.801	0.071	0.236
1.204	0.044	-0.588	0.108	-0.998
0.270	-0.563	-0.398	-0.576	

(a) (15 points) Compute the sample median and forth spread.

Solution:

After ordering the data you obtain

-0.998	-0.993	-0.816	-0.801	-0.628
-0.588	-0.576	-0.563	-0.398	-0.325
0.044	0.071	0.108	0.236	0.270
0.510	0.779	0.970	1.204	

so that:

$$\tilde{x} = x_{10} = -0.325$$

$$lf = \frac{x_5 + x_6}{2} = -0.608 \qquad uf = \frac{x_{14} + x_{15}}{2} = 0.253$$

$$fs = 0.862$$

(b) (10 points) Knowing that $\sum_{i=1}^{19} x_i = -2.493$ and $\sum_{i=1}^{19} x_i^2 = 8.348$ compute the sample mean and standard deviation.

Solution:

$$\bar{x} = \frac{-2.493}{19} = -0.131$$

$$s = \sqrt{\frac{1}{18}\left(8.348 - \frac{2.493^2}{19}\right)} = \sqrt{0.446} = 0.668$$

(c) (15 points) Draw a box plot of the data.

Solution:

See figure below.

Question 2 . . . . *60 point*

A factory produces 1000 computers. Each computer has a probability $p = 0.05$ of having a defect. The factory has a quality control department. If a computer is defective it will be detected and discarded with probability 1. If a computer is not defetective it will be discarded with a probability $s = 0.03$.

(a) (15 points) Let X be the number of defective computers among the 1000 produced. Identify what kind of r.v. is X. Write the p.m.f. of X and compute $E(X)$ and σ_X. This question does not involve the quality control.

Solution: Clearly X is a Binomial r.v. with parameter 1000 and 0.05. It follows that

$$p(x) = P(X = x) = \binom{1000}{x} 0.05^x 0.95^{1000-x}$$

Moreover

$$E(X) = 1000 \cdot 0.05 = 50 \qquad \sigma_X = \sqrt{1000 \cdot 0.05 \cdot 0.95} = \sqrt{47.5} = 6.89$$

(b) (15 points) Use the Normal approximation for a Binomial r.v. to compute the probability that $X > 60$, $P(X > 60)$, and the probability that $X < 30$, $P(X < 30)$. Use the table provided at the end of the text.

Solution: Since X is Binomial with $E(X) = 50$ and $\sigma_X = 6.89$ we know that approxiametely X is $\mathcal{N}(50, 47.5)$. It follows that

$$Z = \frac{X - 50}{6.89}$$

is $\mathcal{N}(0, 1)$. Thus

$$P(X > 60) = P(Z > 1.45) = 1 - \Phi(1.45) = 0.0735$$

while

$$P(X < 30) = P(Z < -2.9) = \Phi(-2.9) = 0.0019$$

(c) (15 points) Compute the probability that a randomly selected computer will be discarded by the quality control department. (**Hint**: Call A the event $\{$*computer is defective*$\}$ and B the event $\{$*computer is discarded*$\}$. You are asked to find $P(B)$. The text gives you $P(A)$, $P(B|A)$ and $P(B|A')$. Use the Law of Total Probability.)

Solution: Call A the event $\{$*computer is defective*$\}$ and B the event $\{$*computer is discarded*$\}$. Then we want to find $P(B)$. We have

$$P(B) = P(B|A)P(A) + P(B|A')P(A') = 1 \cdot 0.05 + 0.03 \cdot 0.95 = 0.079$$

(d) (15 points) Compute the probability that a discarded computer is actually defective. (**Hint**: You are asked to find $P(A|B)$. Use Bayes Theorem or the definition of conditional probability.)

Solution: With the notation of the previous point we want $P(A|B)$. We have

$$P(A|B) = P(B|A)\frac{P(A)}{P(B)} = 1\frac{0.05}{0.079} = 0.64$$

(e) (10 points (bonus)) Call Y the r.v. that describes number of discarded computers that are working. Write the p.m.f. of Y. Using the Poisson approxiamtion, write a formula for the probability that $Y = 20$ (do not try to evaluate it) and the expected value of Y, $E(Y)$.

Solution:
Clearly Y is a Binomial r.v.. with parameters 1000 and p, where p is the probability that a randomly selected computer is both working and discarded. This is given by

$$P(A' \cap B) = P(A')P(B|A') = 0.95 \cdot 0.03 = 0.0285$$

It follows that

$$p_Y(y) = \binom{1000}{y} 0.9715^{1000-y} 0.0285^y.$$

Since $1000 * 0.029 = 29$ we have

$$P(Y = 20) \simeq e^{-28.5}\frac{28.5^{20}}{20!} = 0.0185$$

while

$$E(Y) = 29.$$

3. (10 points (bonus)) Let T be an exponential r.v. with parameter λ. Show that

$$P(T > t + s \,|\, T > s) = P(T > t)$$

Solution: We have

$$P(T > t) = \int_t^\infty \lambda e^{-\lambda t} dt = e^{-\lambda t}$$

Moreover

$$\begin{aligned} P(T > t + s \,|\, T > s) =& \frac{P(T > t + s \text{ and } T > s)}{P(T > s)} = \\ =& \frac{P(T > t + s)}{P(T > s)} = \frac{e^{-\lambda(t+s)}}{e^{-\lambda s}} = e^{-\lambda t} \end{aligned}$$

Useful Formulas

- If A and B are events then the probability of A given B is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

and A and B are independent if $P(A|B) = P(A)$.

- If X is a binomial r.v. with parameters N and p then

$$bin(x; N, p) = P(X = x) = \binom{N}{x} p^x (1-p)^{n-x}.$$

Moreover $E(X) = Np$ and $V(X) = Np(1-p)$.

- If X is a Poisson r.v. with parameter λ then

$$p(x; \lambda) = P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}.$$

Moreover $E(X) = \lambda$ and $V(X) = \lambda$.

- If X is an exponential r.v. with parameter λ then

$$f(x, \lambda) = \lambda e^{-\lambda x}$$

for $x > 0$. Moreover $E(X) = \lambda^{-1}$ and $V(X) = \lambda^{-2}$.

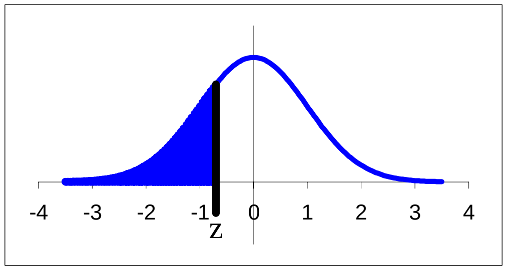
- If Z is a Normal Standard r.v. then $P(Z < z) = \Phi(z)$, see included table. Moreover $\Phi(z) = 1 - \Phi(-z)$.

Table 1a: Standard Normal Probabilities

The values in the table below are cumulative probabilities for the standard normal distribution Z (that is, the normal distribution with mean 0 and standard deviation 1). These probabilities are values of the following integral:

$$P\left(Z \le z\right) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

Geometrically, the values represent the area to the left of z under the density curve of the standard normal distribution:



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641