You can use your book and notes. No laptop or wireless devices allowed. Write clearly and try to make your arguments as linear and simple as possible. The complete solution of one exercise will be considered more that two half solutions.

When returning your Exam, you must return also this page, signed. Thanks.

To solve the Exam problems, I have not collaborated with anyone or used any source except class notes and the textbook.

Name: ______________________________

Question:	1	2	3	4	5	6	Total
Points:	40	10	10	30	30	20	140
Score:							

Question 1 . . . . *40 point*

Your company assembles PCs. A new assembly line is being proposed to your attention. The following data x_i are the result of a random sample of size $N = 50$ of the production time, in minutes, of one PC using the new assembly line. They are ordered in increasing order.

26.16	26.18	26.44	26.78	26.94	27.06	27.15	27.2	27.31	27.39
27.53	27.55	27.70	27.86	28.04	28.23	28.28	28.31	28.47	28.53
28.66	28.80	28.83	28.97	28.97	29.14	29.45	29.59	29.61	29.7
29.73	29.74	29.83	30.21	30.29	30.36	30.48	30.61	30.72	31.21
31.64	31.74	31.89	32.08	32.39	32.39	32.76	32.81	32.91	32.99

You know that

$$\sum_{i=1}^{50} x_i = 1467.84 \qquad \sum_{i=1}^{50} x_i^2 = 43278.96$$

(a) (10 points) Compute the sample average and standard deviation.

(b) (10 points) Compute the median and IQR.

(c) (10 points) After finding eventual outliers, draw a box plot for the data.

(d) (10 points) Choose a reasonable number of classes and draw an histogram for the data, using the densities. Show your computations.

Question 2 . . . . *10 point*

Question 1 continued.

(a) (10 points) Find the Confidence Interval for the true population mean μ at a confidence level of 99%. Use the table at the end of the book to find the necessary critical values.

Question 3 .... *10 point*

The time difference between two consecutive arrivals of a car at a service center is described by the random variable T. The p.d.f. of this variables is:

$$f(t) = \begin{cases} \lambda e^{-\lambda(t-2)} & t > 2 \\ \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The following data are 10 observation t_i of arrival times:

18.138 2.1721 5.43 19.405 3.528 2.9496 2.3909 7.6107 7.7469 19.335.

Use the method of moments and maximum likelihood to compute an estimator for λ based on this sample

Question 4 .... *30 point*

Let X be a discrete r.v. with p.m.f. $p(x)$ given by

x	-2	-1	0	1	2
$p(x)$	0.1	0.2	0.4	0.2	0.1

(a) (10 points) Compute $E(X)$ and $V(X)$.

(b) (10 points) Suppose you have 100 independent r.v. X_i, $i = 1, \ldots, 100$, all with p.m.f given by $p(x)$ above. Give the approximate p.d.f. of $\bar{X} = \frac{1}{100}\sum_{i=1}^{100} X_i$. (**Hint**: Use the CLT.)

(c) (10 points) Compute $P(\bar{X} > 0.1)$.

Question 5 .... *30 point*

A food distributor carries two different brands of a certain type of grain. Its supplies fluctuate randomly in time but never exceed 2 tons. Let X be the amount of brand A on hand and Y the amount of brand B on hand. Suppose that the joint p.d.f. of X and Y is:

$$f(x,y) = \begin{cases} k & x>0,\ y>0,\ x+y<2 \\ \\ 0 & \text{otherwise} \end{cases}$$

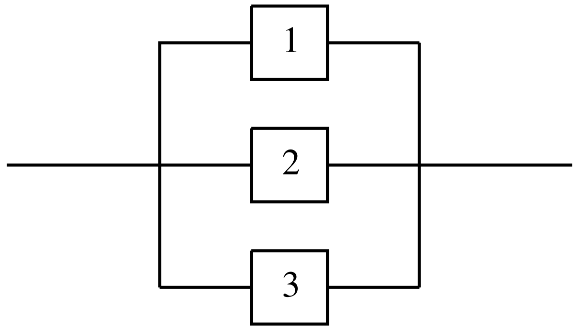
(a) (10 points) Draw the region where $f(x,y) \neq 0$ and find the value of k that makes $f(x,y)$ a p.d.f.

(b) (10 points) Are X and Y independent? Answer by first deriving the marginal p.d.f. of each variable.

(c) (10 points) Find $E(X)$, $E(Y)$, $V(X)$, $V(Y)$ and $\mathrm{corr}(X, Y)$.

Question 6 . . . . *20 point*

Three components are connected in parallel as in the figure below



The lifetime T_1 of the first component is an exponential random variable with parameter 1, the lifetime T_2 of the second component is an exponential random variable with parameter 2 and the lifetime T_3 of the third component is an exponential random variable with parameter 3. The lifetime of the three component are independent. The system fails only if all the components fail. Call T_p the lifetime of the system.

(a) (10 points) Compute the c.d.f of T_p. (**Hint**: clearly $T_p = \max(T_1, T_2, T_3)$.)

(b) (10 points) If at time t you observe that the system is still working what is the probability that the first and second components have both already failed? and the probability that only the first component has failed?(**Hint**: the first proability is the probability that $T_1 < t$ and $T_2 < t$ given that $T_P > t$...)