# Test # 3
# Math 1501, Fall 2008

Dr. Gangbo *

November 9, 2008

**Name:**

**Section:**

**Instructions.** You are to work independently these problems for the next one hour and ten minutes ( 1hr 10mn). You are not allowed to use any textbook, your class notes or a calculator. Read carefully each exercise and show all your work.

*School of Mathematics, Georgia Institute of Technology.

**Exercise 1** (**10 points**). Calculate $F'(x)$

$$F(x) = \int_{-8}^{x^3} t \cos t dt.$$

**Exercise 2** (**15 points**). Assume that $f : \mathbf{R} \to \mathbf{R}$ is continuous and

$$\int_0^x tf(t)dt = \sin x - x \cos x.$$

(a) (**10 points**) Determine $f(\pi/2)$. (b) (**5 points**) Find $f'(x)$.

**Exercise 3** (**10 points**). Find $f$ from the informations

$$f"(x) = \sin x, \quad f'(0) = -2, \quad f(0) = 1.$$

**Exercise 4** (**15 points**). Find the area between the curves

$$y = \cos^2(\pi x), \quad y = \sin^2(\pi x), \quad x = 0, \quad x = \frac{1}{4}.$$

**Exercise 5** (**20 points**). Calculate

$$\int x^{-\frac{3}{4}}\left(x^{\frac{1}{4}}+1\right)^{-2}dx$$

**Exercise 6** (**30 points**). (a) (**5 points**) Recall Hooke's law. (b) (**25 points**) Find the natural length of a spring given that the work required to stretch it from 2 feet to 2.1 feet is one-half of the work required to stretch it from 2.1 feet to 2.2 feet.