Let a_n be the sequence

$$a_n = n\left(\cos\left(\frac{1}{n}\right) - 1\right) \tag{1}$$

Observe that if we call:

$$f(x) = \frac{\cos(x) - 1}{x} \tag{2}$$

and

$$b_n = \frac{1}{n} \tag{3}$$

then we have

$$a_n = f(b_n) \tag{4}$$

We know that, after setting $f(0) = 0$, $f(x)$ is a continuous function for all x real. Moreover $\lim_{n\to\infty} b_n = 0$. Thus it follows from Theorem 11.3.12 that

$$\lim_{n\to\infty} a_n = f\left(\lim_{n\to\infty} b_n\right) = f(0) = 0 \tag{5}$$