No books or notes allowed. No laptop, graphic calculator or wireless devices allowed. Write clearly.

Name: _____

1. (a) (5 points) Let f be the function

$$f(x) = \frac{(x-1)^2}{|x-1|}.$$

Can you define f for $x = 1$ so that f is continuous at 1. (*Hint:* check the right and left limit separatly.)

Solution: For $x > 1$ we have

$$f(x) = (x-1)\frac{x-1}{|x-1|} = x - 1$$

while for $x < 1$

$$f(x) = (x-1)\frac{x-1}{|x-1|} = -(x-1)$$

so that

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = 0$$

Thus setting $f(1) = 0$ we have that $f(x)$ is continuous at 1.

(b) (5 points) Let f be the function

$$f(x) = \frac{x^2 - 1}{|x - 1|}.$$

Can you define f for $x = 1$ so that f is continuous at 1.

Solution: For $x > 1$ we have

$$f(x) = (x + 1)\frac{x - 1}{|x - 1|} = x + 1$$

while for $x < 1$

$$f(x) = (x + 1)\frac{x - 1}{|x - 1|} = -(x + 1)$$

so that

$$\lim_{x \to 1^+} f(x) = 2 \qquad \lim_{x \to 1^-} f(x) = -2$$

Thus $f(x)$ has a jump discontinuity at 1. There is no way to define f for $x = 1$ so that f is continuous at 1.